

Second Edition

ELECTROMAGNETIC FIELDS AND WAVES

Paul Lorrain

Dale R. Corson



W. H. FREEMAN AND COMPANY

TOPPAN COMPANY, LIMITED

Contents

Preface	xix
List of Symbols	xxiii

1

VECTORS 1

1.1	Vector Algebra	2
	Examples	3, 5
1.2	Invariance	5
1.3	The Time Derivative	8
	Example	9
1.4	The Gradient	10
	Example	11
1.5	Flux and Divergence. The Divergence Theorem	13
	Examples	13, 16
1.6	Line Integral and Curl	16
	Examples	16, 19
1.7	Stokes's Theorem	21
	Example	22
1.8	The Laplacian	22
	Example	22

1.9	Curvilinear Coordinates	23
1.9.1	Cylindrical Coordinates	24
1.9.2	Spherical Coordinates	26
1.9.3	The Gradient	27
1.9.4	The Divergence	28
1.9.5	The Curl	30
1.9.6	The Laplacian	31
1.10	Summary	32
	Problems	36

2

ELECTROSTATIC FIELDS I 40

Electrostatic Fields in a Vacuum

2.1	Coulomb's Law	40
	Example	41
2.2	The Electric Field Intensity	42
2.3	The Electric Potential	43
2.4	The Electric Field Inside and Outside Macroscopic Bodies	46
2.5	Gauss's Law	47
	Example: The Average Potential over a Spherical Surface	49
2.6	The Equations of Poisson and of Laplace	51
	Review	51
2.7	Conductors	52
	Example	53
2.8	Calculation of the Electric Field Produced by a Simple Charge Distribution	53
	Example: Field of a Uniform Spherical Charge Distribution	54
2.9	The Electric Dipole	61
2.10	The Linear Electric Quadrupole	64
2.11	Electric Multipoles	65
2.12	The Electric Field Outside an Arbitrary Charge Distribution	66
2.13	The Average Electric Field Intensity Inside a Sphere Containing an Arbitrary Charge Distribution	70
2.14	Potential Energy of a Charge Distribution	72
2.15	Energy Density in an Electric Field	76
	Example	78
2.16	Forces on Conductors	78
	Example	80
	Example: Forces on a Parallel-Plate Capacitor	80
2.17	Summary	81
	Problems	84

3

ELECTROSTATIC FIELDS II 91

Dielectric Materials

- 3.1 The Electric Polarization P 92
- 3.2 Electric Field at an Exterior Point 92
 - 3.2.1 The Bound Charge Densities ρ_b and σ_b 94
 - 3.2.2 The Polarization Current Density 96
- 3.3 Electric Field at an Interior Point 97
 - 3.3.1 Electric Field Intensity E'' due to the Distant Dipoles 98
 - 3.3.2 Electric Field Intensity E''' due to the Near Dipoles 100
 - Review 101
- 3.4 The Local Field 102
- 3.5 The Electric Susceptibility χ_e 104
- 3.6 The Divergence of E . The Electric Displacement D 105
 - 3.6.1 The Relative Permittivity ϵ_r . Poisson's Equation for Dielectrics 108
 - 3.6.2 The Free Charge Density ρ_f and the Bound Charge Density ρ_b 108
- 3.7 Calculation of Electric Fields Involving Dielectrics 109
 - Example:* The Dielectric-Insulated Parallel-Plate Capacitor 110
 - Example:* The Free Charge Density σ_f , the Bound Charge Density σ_b , and the Electric Displacement D at a Dielectric-Conductor Boundary 111
 - Example:* Dielectric Sphere with a Point Charge at its Center 111
 - Example:* The Bar Electret 113
- 3.8 The Clausius-Mossotti Equation 115
- 3.9 Polar Dielectrics 116
 - 3.9.1 The Langevin Equation 117
 - 3.9.2 The Debye Equation 120
- 3.10 Frequency Dependence, Anisotropy, and Nonhomogeneity 121
 - Examples* 122
- 3.11 Potential Energy of a Charge Distribution in the Presence of Dielectrics 123
 - Example:* Energy Stored in a Parallel-Plate Capacitor 124
- 3.12 Forces on Dielectrics 125
 - Example:* Force per Unit Volume on the Insulating Material in a Coaxial Cable 127
- 3.13 Forces on Conductors in the Presence of Dielectrics. Energy Density in Dielectrics 128
 - Example:* Forces on a Parallel-Plate Capacitor Immersed in a Liquid Dielectric 128
- 3.14 Summary 129
 - Problems 131

4

ELECTROSTATIC FIELDS III 138
*General Methods for Solving Laplace's and Poisson's Equations*Continuity of V , D_n , E_t at the Interface Between Two Different Media 139

- 4.1.1 Potential 139
- 4.1.2 Normal Component of the Electric Displacement 139
- 4.1.3 Tangential Component of the Electric Field Intensity 140
- 4.1.4 Bending of Lines of Force 141
- 4.2 The Uniqueness Theorem 142

4.3 Images 144

Example: Point Charge Near an Infinite Grounded Conducting Plane 144*Example:* Point Charge Near a Grounded Conducting Sphere 146*Example:* Point Charge Near a Charged Conducting Sphere 149*Example:* Charged Sphere Near a Grounded Conducting Plane 150*Example:* Charge Near a Semi-infinite Dielectric 153

4.4 Solution of Laplace's Equation in Rectangular Coordinates 156

Example: Field Between Two Grounded Semi-infinite Parallel Electrodes Terminated by a Plane Electrode at Potential V_0 158*Example:* Field Between Two Grounded Parallel Electrodes Terminated on Two Opposite Sides by Plates at Potentials V_1 and V_2 161

4.5 Solution of Laplace's Equation in Spherical Coordinates. Legendre's Equation. Legendre Polynomials 163

Example: Conducting Sphere in a Uniform Electric Field 168*Example:* Dielectric Sphere in a Uniform Electric Field 1734.6 Solution of Poisson's Equation for V 176*Example:* p - n Junction Diode in Silicon 1764.7 Solution of Poisson's Equation for E 180*Examples* 180

4.8 Summary 181

Problems 183

5

RELATIVITY I 193

The Basic Concepts

5.1 The Galilean Transformation 194

5.2 Breakdown of the Galilean Transformation and of Classical Mechanics at High Velocities 195

Example:	Particle Velocities Never Exceed c	195
Example:	The Addition of Velocities	195
Example:	Time Dilation	195
5.3	Inadequacy of the Galilean Transformation for Electromagnetic Phenomena	196
Example:	The Trouton and Noble Experiment	196
Example:	Magnetic Fields	197
Example:	The Jaseva-Javan-Murray-Townes Experiment	198
5.4	The Fundamental Postulate of Relativity	201
5.5	Invariance of a Physical Law as Illustrated by Classical Mechanics	201
Example:	Classical Mechanics Inside a Moving Train	202
Example:	The Law $F = ma$	202
5.6	The Lorentz Transformation	203
5.7	Transformation of a Length	205
Example:	The Apparent Shape of a Rapidly Moving Object	207
5.8	Transformation of a Time Interval	209
Example:	The Time Read on a Rapidly Moving Clock	210
Example:	The Relativistic Doppler Effect for Electromagnetic Waves	212
5.9	Simultaneity	213
5.10	Causality and Maximum Signal Velocity	214
5.11	Transformation of a Velocity	215
Examples		216, 217
5.12	Transformation of an Acceleration	217
5.13	Relativistic Mass	217
5.14	Transformation of a Mass	220
5.15	Relativistic Energy $\mathcal{E}$	220
Example		221
5.16	The Four-Vector $\mathbf{r}$	221
5.17	Transformation of a Momentum and of a Relativistic Energy. The Four-Momentum	223
Example:	The Relation $\mathcal{E}^2 = m_0 c^4 + p^2 c^2$	224
5.18	Transformation of a Force	225
Examples		226
5.19	Transformation of an Element of Volume	226
5.20	Invariance of Electric Charge	228
Example		229
5.21	Transformation of an Electric Charge Density and of an Electric Current. The Four-current Density	229
Examples		231
5.22	The Four-Dimensional Operator $\square$	232
5.23	The Conservation of Charge	233
5.24	Summary	234
Problems		239

6

RELATIVITY II 247

The Electric and Magnetic Fields of Moving Electric Charges

- 6.1 Force Exerted on a Moving Charge by Another Charge Moving at the Same Constant Velocity $\mathbf{v}$ 248
- 6.2 Field of a Charge Moving at a Constant Velocity 251
 - 6.2.1 The Electric Field 256
 - 6.2.2 The Magnetic Field 259
 - Example:* The Field of a 10 GeV Electron at $\theta = 90^\circ$ 260
- 6.3 Transformation of Electric and Magnetic Fields 261
 - Example:* The Parallel-Plate Capacitor 263
- 6.4 The Vector Potential $\mathbf{A}$ 264
 - Example:* The Vector Potential for a 10-GeV Electron at $\theta = 90^\circ$ 266
- 6.5 The Scalar Potential V . The Electric Field Intensity $\mathbf{E}$ Expressed in Terms of V and $\mathbf{A}$ 266
 - Example:* V , ∇V , and $\partial \mathbf{A} / \partial t$ for a 10-GeV Electron 268
- 6.6 Transformation of the Electromagnetic Potentials V and $\mathbf{A}$. The Four-Potential $\mathbf{A}$ 269
 - Example:* The Parallel-Plate Capacitor 270
- 6.7 The Lorentz Condition 271
- 6.8 Gauss's Law 271
 - Example* 272
- 6.9 The Divergence of $\mathbf{B}$ 273
- 6.10 The Curl of $\mathbf{E}$ 273
- 6.11 The Curl of $\mathbf{B}$ 274
- 6.12 Maxwell's Equations 276
 - Example:* The Magnetic Field Near a Straight Wire Carrying a Steady Electric Current 277
 - Example:* The Magnetic Field of a Short Element of Wire Carrying an Electric Current 280
 - Example:* Force on a Second Element $I d\mathbf{l}$ 282
 - Example:* Force on a Moving Charged Particle 283
- 6.13 Summary 283
- Problems 286

7

MAGNETIC FIELDS I 292

Steady Currents and Nonmagnetic Materials

- 7.1 Magnetic Forces 293
- 7.2 The Magnetic Induction $\mathbf{B}$. The Biot-Savart Law 295

Example:	The Magnetic Induction Due to a Current Flowing in a Long Straight Wire	296
Example:	Force Between Two Long Parallel Wires	297
Example:	The Circular Loop	299
7.3	The Force on a Point Charge Moving in a Magnetic Field	299
Example:	Hall Effect in Semiconductors	300
Example:	The Hodoscope	301
7.4	The Divergence of the Magnetic Induction B	302
7.5	The Vector Potential A	303
Example:	The Long Straight Wire	305
Example:	Pair of Long Parallel Wires	306
7.5.1	The Line Integral of the Vector Potential A Over a Closed Curve	308
7.6	The Curl of the Magnetic Induction B	308
7.7	Ampère's Circuital Law	310
Example:	Long Cylindrical Conductor	311
Example:	The Toroidal Coil	313
Example:	The Long Solenoid	315
Example:	Refraction of the Lines of B at a Current Sheet	316
Example:	The Short Solenoid	317
7.8	The Magnetic Dipole	319
Example:	The Long Solenoid	322
7.9	Summary	322
	Problems	323

8

MAGNETIC FIELDS II 332

Induced Electromotance and Magnetic Energy

8.1	The Faraday Induction Law	332
Example:	The Expanding Loop	335
8.1.1	The Faraday Induction Law in Differential Form	336
8.2	The Induced Electric Field Intensity E in Terms of the Vector Potential A	337
Example:	The Electromotance Induced in a Loop by a Pair of Long Parallel Wires Carrying a Variable Current	338
8.3	Induced Electromotance in a Moving System	339
Example:	The Electromotance Induced in a Fixed Loop in a Time-dependent Magnetic Field	342
Example:	The Electromotance Induced in a Loop Rotating in a Fixed Magnetic Field	342
8.4	Inductance and Induced Electromotance	343
8.4.1	Mutual Inductance	343
8.4.2	Self-inductance	345

Example:	Self-inductance of a Long Solenoid	346
Example:	Self-inductance of a Toroidal Coil	347
Example:	Mutual Inductance Between Two Coaxial Solenoids	348
8.4.3	Coefficient of Coupling	350
Example		351
8.5	Energy Stored in a Magnetic Field	351
Example		352
8.5.1	Magnetic Energy in Terms of the Magnetic Induction B	353
Example		354
8.5.2	Magnetic Energy in Terms of the Current Density J , and of the Vector Potential A	354
Examples		355
8.5.3	Magnetic Energy in Terms of the Current I and of the Magnetic Flux Φ	355
Examples		355
8.5.4	Magnetic Energy in Terms of the Currents and of the Inductances	355
Examples		356
8.6	Self-inductance for a Volume Distribution of Current	356
Example:	Self-inductance of a Coaxial Line	356
8.7	Magnetic Force Between Two Circuits	358
8.7.1	The Magnetic Force When the Currents are Kept Constant	359
Example:	Force Between Two Coaxial Solenoids	361
8.7.2	Magnetic Force When the Fluxes Are Kept Constant	363
Example:	Force Between Two Coaxial Solenoids	363
8.8	Magnetic Torque	364
Example:	Magnetic Torque on a Current Loop	365
8.9	Magnetic Forces Within an Isolated Circuit	366
Example:	The Pinch Effect	366
8.10	Magnetic Pressure	367
Example:	Magnetic Pressure Inside a Long Solenoid	369
8.11	Summary	370
Problems		373

9

MAGNETIC FIELDS III 383

Magnetic Materials

9.1	The Magnetization M	384
9.2	Magnetic Induction B at an Exterior Point	384
9.3	Magnetic Induction B at an Interior Point. The Divergence of B	387
Example:	The B Field of a Uniformly Magnetized Cylinder	393

9.4	The Magnetic Field Intensity H . Ampère's Circuital Law	393
9.5	Magnetic Susceptibility χ_m and Relative Permeability μ_r	395
9.5.1	The Equivalent Current Density $\nabla \times M$ and the Free Current Density J_f	396
9.6	Hysteresis	396
9.6.1	Energy Dissipated in a Hysteresis Cycle	398
	Example	400
9.7	Boundary Conditions	400
9.8	Magnetic Field Calculations	402
	Example: The B and H Fields of a Bar Magnet	402
	Example: The Bar Magnet and the Bar Electret	404
9.8.1	Magnetic Circuits	405
9.8.1.1	Magnetic Circuit With an Air Gap	407
	Example	409
9.8.1.2	Magnetic Circuit Energized by a Permanent Magnet	409
	Example	411
9.8.2	Solution of Poisson's Equation for B	413
	Example: The Magnetic Induction at the Center of a Rotating Disk of Charge	413
9.9	Summary	414
	Problems	417

10

MAXWELL'S EQUATIONS 422

10.1	The Conservation of Electric Charge	422
	Example: Charge Density in a Conductor	424
10.2	The Potentials V and A	424
10.2.1	The Retarded Potentials	427
	Example: The Retarded Potentials for an Oscillating Electric Dipole	427
	Example: The Retarded Potential A for an Oscillating Magnetic Dipole	428
10.3	The Lorentz Condition	432
	Example: The Leaky Spherical Capacitor	434
10.4	The Divergence of E and the Nonhomogeneous Wave Equation for V	436
10.5	The Nonhomogeneous Wave Equation for A	437
10.6	The Curl of B	437
	Example: The Displacement Current Density in a Conductor	439
10.7	Maxwell's Equations	439
10.7.1	Maxwell's Equations in Integral Form	442

10.8	Duality	444
	<i>Examples</i>	446
10.9	Lorentz's Lemma	446
	<i>Examples</i>	445, 446
10.10	The Nonhomogeneous Wave Equations for E and B	448
10.11	Summary	450
	Problems	453

11

PROPAGATION OF ELECTROMAGNETIC WAVES I 459

Plane Waves in Infinite Media

11.1	Plane Electromagnetic Waves in Free Space	460
11.1.1	The Poynting Vector	464
	<i>Examples</i>	465
	<i>Example:</i> Energy Flow Through an Imaginary Cylinder	465
11.2	The E and H Vectors in Homogeneous, Isotropic, Linear, and Stationary Media	467
11.3	Propagation of Plane Electromagnetic Waves in Nonconductors	470
	<i>Example</i>	471
11.4	Propagation of Plane Electromagnetic Waves in Conducting Media	471
11.4.1	The Poynting Vector in Conducting Media	473
11.5	Propagation of Plane Electromagnetic Waves in Good Conductors	475
	<i>Example:</i> Propagation in Copper at 1 Megahertz	479
	<i>Example:</i> Joule Losses in Good Conductors	480
11.6	Propagation of Plane Electromagnetic Waves in Low-pressure Ionized Gases	481
	<i>Example</i>	483
11.6.1	The Conductivity σ of an Ionized Gas	483
	<i>Example</i>	485
11.6.2	The Plasma Angular Frequency ω_p	486
11.6.3	Wave Propagation at High Frequencies where $\omega > \omega_p$	489
11.6.4	Wave Propagation at Low Frequencies where $\omega < \omega_p$	491
	<i>Examples</i>	492
	<i>Example:</i> The Ionosphere	492
11.7	Summary	493
	Problems	495

12

PROPAGATION OF
ELECTROMAGNETIC WAVES II 504*Reflection and Refraction*

- 12.1 The Laws of Reflection and Snell's Law of Refraction 505
- 12.2 Fresnel's Equations 508
 - 12.2.1 Incident Wave Polarized with its E Vector *Normal* to the Plane of Incidence 508
 - 12.2.2 Incident Wave Polarized with its E Vector *Parallel* to the Plane of Incidence 510
- 12.3 Reflection and Refraction at the Interface Between Two Nonmagnetic Nonconductors 511
 - 12.3.1 The Brewster Angle 515
 - Example:* Measuring the Relative Permittivity of the Moon's Surface at Radio Frequencies 517
 - 12.3.2 The Coefficients of Reflection and of Transmission at an Interface Between Two Nonconductors 517
- 12.4 Total Reflection at an Interface Between Two Nonconductors 520
 - Example:* Light Emission from a Cathode Ray Tube 525
 - Example:* The Critical Angle and Brewster's Angle 526
 - 12.4.1 Demonstration of the Validity of Snell's Law, of the Laws of Reflection and Refraction, and of Fresnel's Equations in the Case of Total Reflection 527
 - 12.4.1.1 The Wave Numbers k_{1x} and k_{1z} for the Reflected Wave 528
 - 12.4.1.2 The Wave Numbers k_{2x} and k_{2z} for the Transmitted Wave 528
 - 12.4.1.3 The Amplitudes of E and H in the Reflected and Transmitted Waves 529
 - 12.4.1.4 The Poynting Vector for the Transmitted Wave 531
- 12.5 Reflection and Refraction at the Surface of a Good Conductor 532
 - Example:* Communicating with Submarines at Sea 535
 - Example:* Standing Waves at Normal Incidence 535
 - Example:* Transmission of an Electromagnetic Wave Through a Thin Sheet of Copper at Normal Incidence 536
 - 12.5.1 Demonstration of the Validity of Snell's Law, of the Laws of Reflection and Refraction, and of Fresnel's Equations at the Interface Between a Nonconductor and a Good Conductor 540
 - 12.5.1.1 The Wave Numbers k_{2x} and k_{2z} for Refraction in a Good Conductor 541
 - 12.5.1.2 The Amplitudes of E and H in the Reflected and Transmitted Waves 542
- 12.6 Radiation Pressure at Normal Incidence on a Good Conductor 543
 - Examples* 546
- 12.7 Reflection of an Electromagnetic Wave by an Ionized Gas 547
- 12.8 Summary 549
- Problems 551

13

PROPAGATION OF
ELECTROMAGNETIC WAVES III 557*Guided Waves*

13.1	Propagation in a Straight Line	557
13.1.1	TE and TM Waves	560
13.1.2	TEM Waves	561
13.1.3	Boundary Conditions at the Surface of Metallic Wave Guides	563
13.2	The Coaxial Line	566
13.3	The Hollow Rectangular Wave Guide	568
13.3.1	The TE Wave	569
13.3.2	Internal Reflections	575
13.3.3	Energy Transmission	577
13.3.4	Attenuation	578
13.4	Summary	582
	Problems	585

14

RADIATION OF ELECTROMAGNETIC WAVES 595

14.1	Electric Dipole Radiation	595
14.1.1	The Scalar Potential V	597
14.1.2	The Vector Potential A and the Magnetic Field Intensity H	598
14.1.3	The Electric Field Intensity E	601
14.1.4	The Average Poynting Vector and the Radiated Power	603
14.1.5	The Electric and Magnetic Lines of Force	605
14.1.6	The $K\lambda$ Surface	607
14.2	Radiation from a Half-wave Antenna	611
14.2.1	The Electric Field Intensity E	611
14.2.2	The Magnetic Field Intensity H	614
14.2.3	The Average Poynting Vector and the Radiated Power	614
14.3	Antenna Arrays	616
14.4	Electric Quadrupole Radiation	620
14.5	Magnetic Dipole Radiation	623
14.5.1	The Potentials V and A	624
14.5.2	The E and H Vectors	624
14.5.3	The Average Poynting Vector and the Radiated Power	625
14.6	Magnetic Quadrupole Radiation	626
14.7	The Electric and Magnetic Dipoles as Receiving Antennas	627
14.8	The Reciprocity Theorem	629
14.9	Summary	633
	Problems	635

APPENDIXES 641

A. Conversion Table	643
B. The Complex Potential	644
B.1 Functions of the Complex Variable	644
B.2 Conformal Transformations	646
B.3 The Function $W(z)$ as a Complex Potential	647
<i>Example:</i>	648
B.4 The Stream Function	649
<i>Example:</i> The Parallel-plate Capacitor	651
<i>Example:</i> The Cylindrical Capacitor	651
<i>Example:</i> Field of Two Parallel Line Charges of Opposite Polarities	653
<i>Example:</i> Field of Two Parallel Conducting Circular Cylinders of Opposite Polarities	654
Problems	656
C. Induced Electromotance in Moving Systems	657
<i>Example:</i> Experiment 1	657
<i>Example:</i> Experiment 2	660
<i>Example:</i> Experiment 3	662
D. The Exponential Notation	665
D.1 The $j\omega$ Operator	665
<i>Example:</i> Solving a Linear Differential Equation with Constant Coefficients, Using the Exponential Notation	668
E. Waves	670
E.1 Plane Sinusoidal Waves	670
E.2 Waves on a Stretched String. The Differential Equation for an Unattenuated Wave	674
E.3 Solution of the Differential Equation for an Unattenuated Wave by the Separation of Variables	676
E.4 Reflection of a Wave on a Stretched String at a Point Where the Density Changes from ρ_1 to ρ_2	677
E.5 Waves on a Stretched String with Damping The Differential Equation for an Attenuated Wave	679
E.6 Solution of the Differential Equation for an Attenuated Wave by the Separation of Variables	681
E.7 Wave Propagation in Three Dimensions	682
E.8 Wave Propagation of a Vector Quantity	683
E.9 The Nonhomogeneous Wave Equation	683

ANSWERS 685

INDEX 699

Preface

This is a revised version of *Introduction to Electromagnetic Fields and Waves* by the same authors. The general level is unchanged, despite the fact that the phrase "Introduction to" has been omitted in the title.

Like the first edition, this book is intended primarily for students who have had at least one full course in electricity and magnetism, and one full course in calculus, including an introduction to differential equations. It should also be useful for scientists and engineers who wish to review the subject.

The major change has been the addition of two chapters on relativity. *Those who are pressed for time may omit these without losing continuity.* The other chapters have been largely rewritten, with many additions and deletions. There are about 100 new figures; as in the first edition, three-dimensional objects and phenomena are represented as such, and the field maps (such as those in Chapter 4) have been plotted on a computer. Most of the 140 examples and most of the 413 problems are new.

The aim of this book is to give the reader a working knowledge of the basic concepts of electromagnetism. Indeed, as Alfred North Whitehead stated, half a century ago, "Education is the acquisition of the art of the *utilization* of knowledge." This explains the relatively large number of examples and problems. It also explains why we have covered fewer subjects more thoroughly. For instance, Laplace's equation is solved in rectangular and in spherical coordinates, but not in cylindrical coordinates.

CONTENTS

So as to reduce the mathematical requirements, we have included a chapter on vectors (Chapter 1), a discussion of Legendre's differential equation