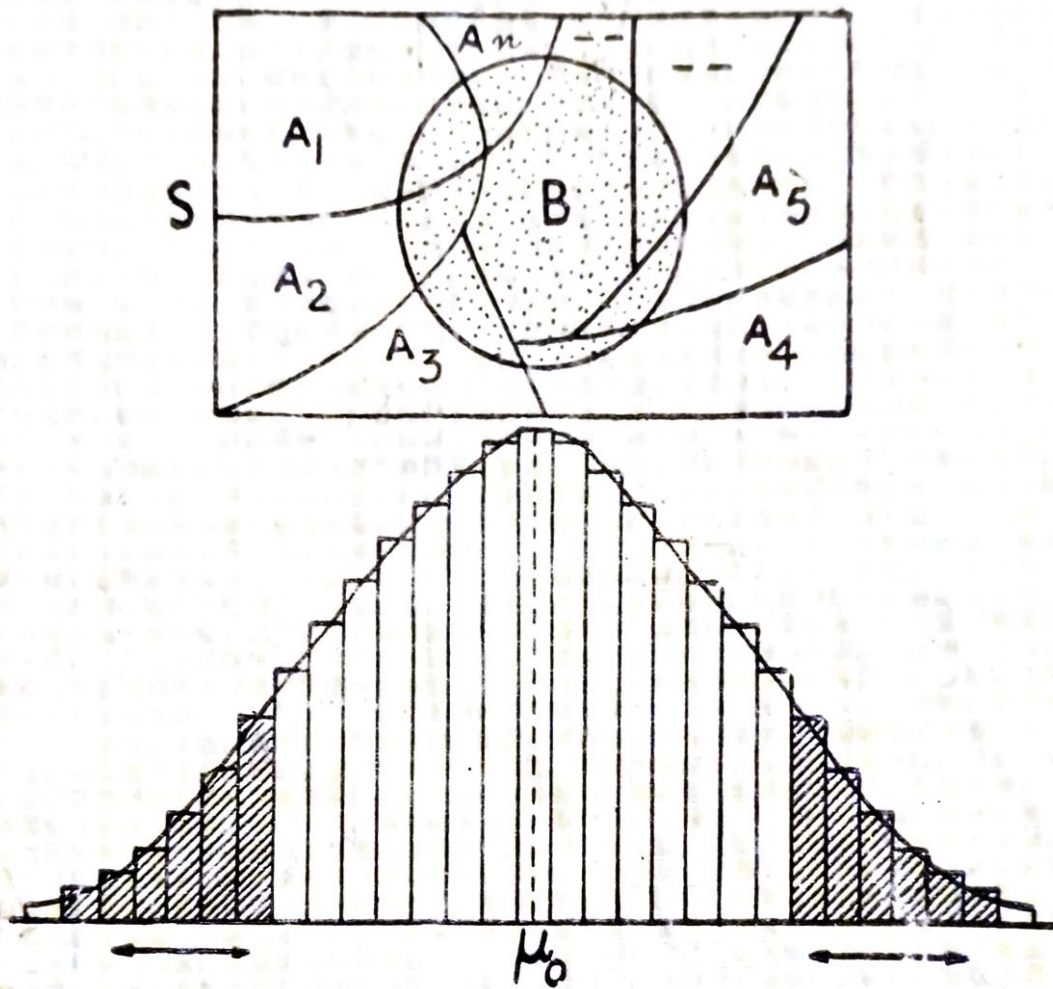


Introduction to STATISTICAL THEORY

PART I



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ILMI KITAB KHANA

I-URDU BAZAR, LAHORE - 2 (W.P.)

INTRODUCTION TO STATISTICAL THEORY

PART I

(A text book for B.A./B.Sc. Pass & Hons. Students)

BY

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CONTENTS



<i>Preface to Fourth Edition</i>	v
<i>Preface to Second Edition</i>	v
<i>Preface to First Edition</i>	vii

I INTRODUCTION

1.1. Meaning of Statistics	1
1.1.1. Main Divisions of Statistics	1
1.1.2. Importance of Statistics	2
1.1.3. Limitations of Statistics	2
1.1.4. Characteristics of Statistics	3
1.2. Qualitative and Quantitative Variables	3
1.2.1. Continuous and Discrete Variables	3
1.2.2. Errors of Measurements	4
1.2.3. Significant Digits	4
1.2.4. Rounding off a Number	5
1.3. Collection of Data	6
1.3.1. Collection of Primary Data	6
1.3.2. Collection of Secondary Data	7
1.3.3. Editing of Data	8
Exercises	8

2 PRESENTATION OF DATA

2.1. Introduction	10
2.2. Classification	10
2.3. Tabulation	10
2.3.1. Types of Tables	11
2.3.2. Main Steps in Tabulation	11
2.4. Frequency Distribution and its Formation	19
2.4.1. Class-limits	19
2.4.2. Class-boundaries	19
2.4.3. Midpoint	19
2.4.4. Class-Interval	20
2.4.5. Making a Frequency Distribution	20
2.4.6. Cumulative Distributions	24
2.5. Graphical Representation	25

2.5.1. Diagrams	...	PAGE
2.5.1.1. Simple Bar Chart	...	25
2.5.1.2. Multiple Bar Chart	...	27
2.5.1.3. Component Bar Chart	...	28
2.5.1.4. Rectangles and Sub-divided Rectangles	...	29
2.5.1.5. Squares	...	30
2.5.1.6. Three-Dimensional Diagrams or Cubes	...	32
2.5.1.7. Pictograms	...	33
2.5.1.8. Pie Diagrams	...	34
2.5.1.9. Profit and Loss Chart	...	35
2.5.2. Graphs	...	36
2.5.2.1. Construction of Graphs	...	37
2.5.2.2. Graph of Time Series-Histogram	...	38
2.5.2.3. Histogram	...	40
2.5.2.4. Frequency Polygon	...	42
2.5.2.5. Frequency Curve	...	43
2.5.2.6. Cumulative Frequency Polygon or Ogive	...	43
2.5.2.7. Types of Frequency Curves	...	45
2.5.3. Ratio Charts or Semi-logarithmic Graphs	...	47
Exercises	...	48
3 MEASURES OF CENTRAL TENDENCY OR AVERAGES		
3.1. Definition	...	60
3.2. Desirable Qualities of a Good Average	...	60
3.3. Types of Averages	...	60
3.4. The Arithmetic Mean	...	61
3.4.1. Change of Origin and Unit	...	61
3.4.2. The Weighted Arithmetic Mean	...	62
3.4.3. Properties of the Arithmetic Mean	...	67
3.5. The Geometric Mean	...	68
3.6. The Harmonic Mean	...	70
3.7. The Median	...	74
3.7.1. Quantiles	...	75
3.8. The Mode	...	79
3.9. Empirical Relation Between Mean, Median and Mode	...	81
3.10. Relative Merits and Demerits of Various Averages	...	81

	PAGE
3.10.1. Arithmetic Mean	81
3.10.2. Geometric Mean	82
3.10.3. Harmonic Mean	82
3.10.4. Median	83
3.10.5. Mode	83
Exercises	83
4 MEASURES OF DISPERSION, MOMENTS AND SKEWNESS	
4.1. Introduction	93
4.2. Range	93
4.3. Semi-Interquartile Range or Quartile Deviation	94
4.4. The Mean Deviation	96
4.5. Standard Deviation	99
4.5.1. Change of Origin and Unit	100
4.5.2. Relation between the Standard deviation and the Root-mean-square deviation	103
4.5.3. Variance	104
4.5.4. Co-efficient of Variation	105
4.5.5. Standard Measure	109
4.6. Moments	109
4.6.1. Moments about the Mean in terms of Moments about any other value, say a , and conversely	110
4.6.2. Moment-Ratios	112
4.6.3. Sheppard's Corrections	112
4.6.4. Change of Origin and Scale	114
4.6.5. Charlier Check	118
4.6.6. Factorial Moments	120
4.7. Skewness	121
4.8. Kurtosis	122
4.9. Interpretation of Frequency Distributions	123
Exercises	123
5 INDEX NUMBERS	
5.1. Definition	136
5.2. Type of Index Numbers	136
5.3. Main Steps in the Construction of Index Numbers of Prices	136
5.3.1. Selection of items for Inclusion	137
5.3.2. Selection of the Base Period	137
5.3.3. Method of Averaging	139

	PAGE
5.3.4. Selection of Suitable Weights	144
5.4. Some Weighted Index Numbers	144
5.4.1. Laspeyres' Index	144
5.4.2. Paasche's Index	146
5.4.3. Marshall-Edgeworth Index	146
5.4.4. Walsh Index	146
5.4.5. Palgrave's Index	146
5.4.6. Fisher's "Ideal" Index	147
5.5. Quantity Index Numbers	151
5.6. Theoretical Tests for Index Numbers	155
5.6.1. Time Reversal Test	155
5.6.2. Factor Reversal Test	156
5.6.3. Circular Test	159
5.7. Cost of Living Index Numbers	160
5.7.1. Meaning	160
5.7.2. Construction of cost of Living Index Numbers...	160
5.7.3. Shortcomings or Drawbacks of Cost of Living Index Numbers	165
5.8. Use of Index Numbers	165
5.9. Limitations	166
Exercises	167
6 METHOD OF LEAST SQUARES AND CURVE FITTING	
6.1. The Principle of Least Squares	178
6.1.1. Application	178
6.1.2. Deduction of Normal Equations	178
6.2. Curve Fitting	180
6.2.1. Fitting of a Straight Line	180
6.2.2. Fitting of Parabolic or Higher Degree Curves...	182
6.2.3. Change of Origin and Unit	184
6.3. Exponential Curves	185
6.3.1. Modified Exponential Curve	188
6.3.2. Other Types of Curves	189
6.3.3. Criteria for a Suitable Curve	190
Exercises	190
7 SIMPLE REGRESSION AND CORRELATION	
7.1. Regression	197
7.1.1. Equations to the Lines of Regression	197
7.2. Correlation	200

7.2.1.	Scatter Diagram	...	200
7.2.2.	Co-efficient of Correlation	...	201
7.2.3.	Properties of the Correlation Co-efficient r	...	202
7.2.4.	Correlation Table	...	205
7.3.	Rank Correlation	...	211
7.4.	Co-efficient of Concordance	...	213
7.5.	The Correlation Ratio	...	214
7.6.	Biserial Correlation	...	217
7.7.	Tetrachoric Correlation	...	217
	Exercises	...	218

8 PARTIAL AND MULTIPLE CORRELATION

8.1.	Introduction	...	230
8.2.	Yule's Notation	...	230
8.3.	Multiple Regression	...	231
8.3.1.	Properties of the Residuals	...	234
8.3.2.	Variance of a Residual	...	235
8.4.	Multiple Correlation Co-efficient	...	236
8.5.	Partial Correlation Co-efficient	...	239
8.5.1.	Relationship between Multiple and Partial Correlation Co-efficients	...	242
8.5.2.	Reduction Formula for the Order of a Regression Co-efficient	...	242
	Exercises	...	243

9 ANALYSIS OF TIME SERIES

9.1.	Time Series	...	250
9.2.	Main Components of a Time Series	...	250
9.3.	Analysis of Time Series	...	251
9.4.	Measurement of Secular Trend	...	251
9.4.1.	The Freehand Curve Method	...	252
9.4.2.	The Method of Semi-Averages	...	252
9.4.3.	The Moving Average Method	...	252
9.4.4.	The Method of Least Squares	...	253
9.5.	Seasonal Variations	...	260
9.5.1.	Seasonal Averages and Seasonal Indices	...	260
9.5.2.	Computation of Seasonal Indices	...	260
9.6.	Deseasonalization of Data	...	268
9.7.	Measurement of Cyclical and Irregular Variations	...	268
9.8.	Correlation between Time Series	...	269
	Exercises	...	269

	PAGE
10 PROBABILITY THEORY AND GENERATING FUNCTIONS	
10.1 Set Theory	277
10.1.1. Sets	277
10.1.2. Subsets	278
10.1.3. Venn Diagram	278
10.1.4. Set Operations	279
10.1.5. Class of Sets	280
10.1.6. Product Sets	280
10.1.7. Algebra of Sets	281
10.2. Probability Theory	282
10.2.1. Sample Space and Event	282
10.2.2. Definitions of Probability	283
10.2.3. Notations	285
10.3. Laws of Probability	285
10.3.1. Addition Law for Mutually Exclusive Events	285
10.3.2. Complement of an Event	288
10.3.3. If $A \subset B$, then $P(A) \leq P(B)$	289
10.3.4. Addition Law for Not-Mutually Exclusive Events	289
10.3.5. Multiplication Law for Independent Events	290
10.3.6. Multiplication Law for Not-Independent Events	291
10.3.7. Repeated or Independent Trials with Two Outcomes	297
10.3.8. Bayes' Theorem	298
10.4. Random Variable and Mathematical Expectation	300
10.4.1. Expectation of a Constant	301
10.4.2. Expectation of a Sum of Random Variables	301
10.4.3. Expectation of a Product of Independent Random Variables	303
10.4.4. Covariance of Independent Random Variables	303
10.5. Generating Functions	303
10.5.1. Probability Generating Function	303
10.5.2. Moment Generating Function	304
10.5.3. Cumulant Generating Function	305
10.5.4. Relation between Cumulants and Moments	306
10.5.5. Characteristic Function	307
Exercises	307

11 DISCRETE PROBABILITY DISTRIBUTIONS

11.1. The Binomial Probability Distribution	...	319
11.1.1. Derivation	...	319
11.1.2. The Binomial Frequency Distribution	...	322
11.1.3. First Four Moments of the Binomial Distribution	...	322
11.1.4. Moment Generating and Cumulant Generating Functions of the Binomial Distribution	..	325
11.1.5. Fitting a Binomial Distribution to Observed Data	...	327
11.2. The Poisson Distribution	...	328
11.2.1. Derivation	...	329
11.2.2. The Poisson Frequency Distribution	...	331
11.2.3. First Four Moments of the Poisson Distribution	...	332
11.2.4. Moment Generating and Cumulant Generating Function of the Poisson Distribution	...	333
11.2.5. Reproductive Property of the Poisson Distribution	...	334
11.2.6. Fitting a Poisson Distribution to Observed Data	...	336
11.3. Other Discrete Probability Distributions	...	337
11.3.1. The Uniform Distribution	...	337
11.3.2. The Multinomial Distribution	...	338
11.3.3. The Negative Binomial Distribution	...	339
11.3.4. The Geometric Distribution	...	341
11.3.5. The Hypergeometric Distribution	...	342
Exercises	...	345

12 CONTINUOUS PROBABILITY DISTRIBUTIONS

12.1. Introduction	...	353
12.1.1. Probability Density Function	...	353
12.1.2. Distribution Function	...	354
12.1.3. Definitions of Moments, etc., for Continuous Probability Distribution	...	354
12.1.4. Chebyshev's Inequality	...	361
12.2. Continuous Bivariate Distributions	...	363
12.2.1. Marginal Distribution	...	364