

Basics of
**Computer
Aided
Geometric
Design**

An Algorithmic
Approach

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Basics of Computer Aided Geometric Design An Algorithmic Approach

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