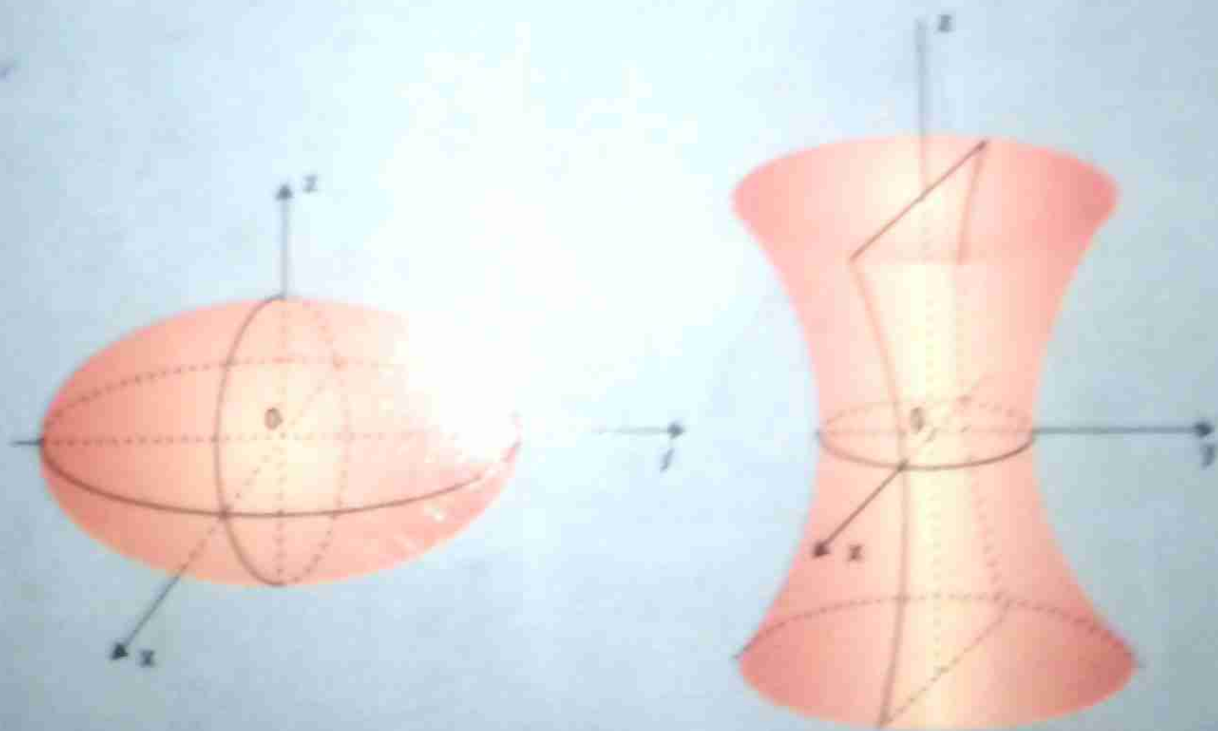


S.M. YUSUF
MUHAMMAD AMIN

CALCULUS WITH ANALYTIC GEOMETRY

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ILMI KITAB KHANA

Kabir Street, Urdu Bazar, Lahore-54000

CALCULUS WITH ANALYTIC GEOMETRY

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