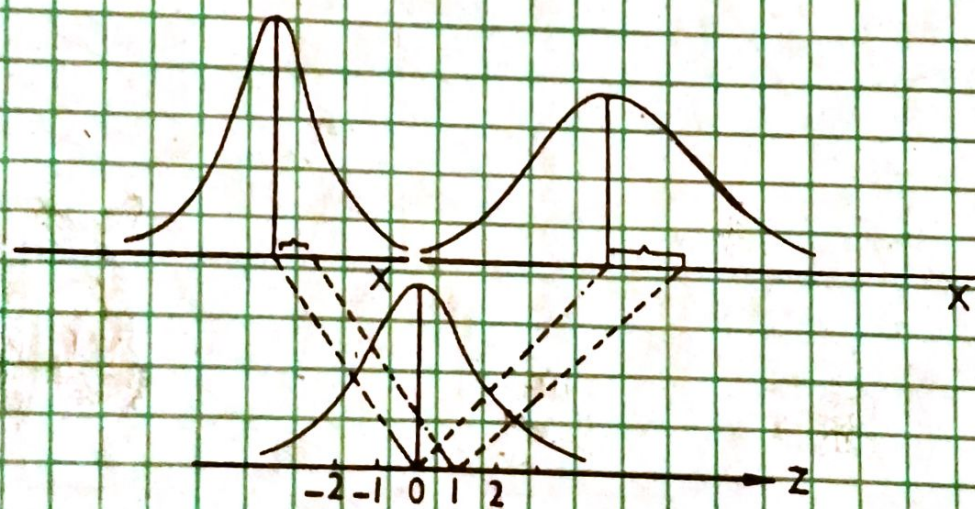


PART 1

DR. S. G. Kulkarni



ILMI KITAB KHANA

Kabir Street, Urdu Bazar, Lahore. 54000



MATHS/AMR/DB

Professor Sher Mohammad Chowdhry,
Head of Department of Statistics,
Government College,
Lahore.
(WEST PAKISTAN).

THE UNIVERSITY OF ASTON IN BIRMINGHAM

Gosta Green, Birmingham B4 7ET / Tel: 021.359 3811 Ex

The Department of Mathematics
Head of Department: Professor N Mullineux
Professor D F Lawden

24th February 1972

Dear Professor Sher Mohammad Chowdhry,

I am very thankful to you for a gift of your books which I received a few days ago. I have found the books very well written and of great use to the student community and would like to congratulate you for producing such standard text-books on "statistics".

With kind regards.

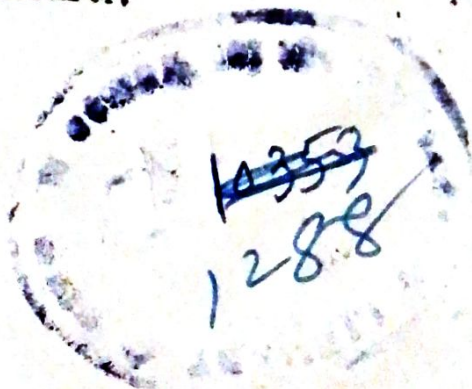
SMB

$\frac{2}{6} - \frac{1}{2}$

Yours sincerely, -

A.M. Rajput

A.M. Rajput.
Senior Lecturer.



CONTENTS

		<u>Page</u>
	Preface	
1.	INTRODUCTION	
1.1	Meaning of Statistics	1
1.1.1	Characteristics of Statistics	2
1.1.2	Descriptive and Inferential Statistics	2
1.1.3	Populations and Samples	3
1.1.4	Importance of Statistics	3
1.2	Observations and Variables	4
1.2.1	Variables	4
1.2.2	Discrete and Continuous Variables	4
1.2.3	Measurement Scales	5
1.2.4	Errors of Measurement	6
1.2.5	Significant Digits	7
1.2.6	Rounding off a Number	8
1.3	Collection of Data	8
1.3.1	Collection of Primary Data	9
1.3.2	Collection of Secondary Data	10
1.3.3	Editing of Data	10
	Exercises	11
2.	PRESENTATION OF DATA	
2.1	Introduction	13
2.2	Classification	13
2.2.1	Aims of Classification	13
2.2.2	Basic Principles of Classification	14
2.3	Tabulation	14
2.3.1	Types of Tables	14
2.3.2	Main Parts of a Table and Construction	14
2.4	Frequency Distribution	19
2.4.1	Class-limits	20
2.4.2	Class-boundaries	20
2.4.3	Class-Mark	21
2.4.4	Class Width or Interval	21
2.4.5	Constructing a Grouped Frequency Distribution	21
2.4.6	Cumulative Frequency Distribution	26
2.5	Stem-and-Leaf Display	27
2.6	Graphical Representation	29
2.7	Diagrams	30
2.7.1	Simple Bar Chart	31
2.7.2	Multiple Bar Chart	32
2.7.3	Component Bar Chart	32

2.7.4	Rectangles and Sub-divided Rectangles	...	33
2.7.5	Pictograms	...	35
2.7.6	Pie Diagrams	...	37
2.7.7	Profit and Loss Chart	...	37
2.8	Graphs	...	39
2.8.1	Graph of Time Series—Hisorigram	...	39
2.8.2	Histogram	...	41
2.8.3	Frequency Polygon	...	42
2.8.4	Frequency Curve	...	43
2.8.5	Cumulative Frequency Polygon or Ogive	...	43
2.8.6	Ogive for Discrete Variable	...	44
2.8.7	Types of Frequency Curves	...	45
2.8.8	Ratio Charts or Semi-logarithmic Graphs...	...	46
	Exercises		

3. MEASURES OF CENTRAL TENDENCY OR AVERAGES

3.1	Introduction	...	57
3.2	Criteria of a Satisfactory Average	...	58
3.3	Types of Averages	...	58
3.4	The Arithmetic Mean	...	59
3.4.1	The Weighted Arithmetic Mean	...	60
3.4.2	Properties of the Arithmetic Mean	...	62
3.4.3	Mean from Grouped Data	...	64
3.4.4	Change of Origin and Scale	...	66
3.5	The Geometric Mean	...	68
3.6	The Harmonic Mean	...	71
3.7	The Median	...	73
3.7.1	Quantiles	...	77
3.8	The Mode	...	78
3.9	Empirical Relation between Mean, Median and Mode	...	79
3.10	The Box Plots	...	80
3.11	Relative Merits and Demerits of Various Averages	...	80
3.11.1	The Arithmetic Mean	...	80
3.11.2	The Geometric Mean	...	80
3.11.3	The Harmonic Mean	...	80
3.11.4	The Median	...	81
3.11.5	The Mode	...	81
	Exercises	...	81

4. MEASURES OF DISPERSION, MOMENTS AND SKEWNESS

4.1	Introduction		
4.2	Range	...	93
		...	94

4.3	The Semi-Interquartile Range or the Quartile Deviation	95
4.4	The Mean (or Average) Deviation	96
4.5	The Variance and Standard Deviation	98
4.5.1	Change of Origin and Scale	103
4.5.2	Interpretation of the Standard Deviation	105
4.5.3	Co-efficient of Variation	105
4.5.4	Properties of Variance and Standard Deviation	108
4.5.5	Standardized Variables	111
4.6	Trimmed and Winsorized Measures	112
4.7	Moments	113
4.7.1	Moments about the Mean in terms of Moments about an arbitrary origin, say a, and conversely	114
4.7.2	Sheppard's Corrections	116
4.7.3	Moment-Ratios	117
4.7.4	Change of Origin and Scale	119
4.7.5	Charlier Check	119
4.8	Skewness	123
4.9	Kurtosis	124
4.10	Describing a Frequency Distribution Exercises	125
		126
5.	INDEX NUMBERS	
5.1	Introduction	
5.1.1	Simple and Composite Index Numbers	139
5.1.2	Problems Involved in Index Number Construction	139
5.2	Main Steps in the Construction of Index Numbers of Wholesale Prices	140
5.2.1	Selection of Commodities for Inclusion	140
5.2.2	Selection of the Base Period	141
5.2.3	Selection of Average	141
5.2.4	Selection of Appropriate Weights	143
5.3	Unweighted Index Numbers	144
5.3.1	Simple Aggregative Index	145
5.3.2	Simple Average of Relatives	145
5.4	Weighted Index Numbers	151
5.4.1	Weighted Aggregative Price Index Numbers	151
5.4.2	Weighted Average of Relatives Price Index Number	158
5.5	Quantity Index Numbers	158
5.6	Tests for Index Number Formulae	163
5.6.1	Time Reversal Test	163
5.6.2	Factor Reversal Test	164
5.6.3	Circular Test	167

5.7	Consumer Price Index Number	...	168
5.7.1	Meaning	...	168
5.7.2	Construction of Consumer Price Index Number	...	168
5.7.3	Shortcomings or Drawbacks of Consumer Price Index Numbers	...	172
5.8	Uses of Index Numbers	...	172
5.9	Limitations of Index Numbers	...	173
	Exercises	...	174
6.	PROBABILITY		
6.1	Introduction	...	185
6.2	An Aside - Sets	...	186
6.2.1	Subsets	...	187
6.2.2	Venn Diagram	...	188
6.2.3	Operations on Sets	...	188
6.2.4	The Algebra of Sets	...	190
6.2.5	Partition of Sets	...	191
6.2.6	Class of Sets	...	191
6.2.7	Cartesian Product Sets	...	191
6.2.8	Relation and Function	...	192
6.3	Random Experiment	...	193
6.3.1	Sample Space	...	193
6.3.2	Events	...	194
6.3.3	Events and Symbolic Representations	...	196
6.3.4	Counting Sample Points	...	196
6.4	Definitions of Probability	...	200
6.4.1	Subjective or Personalistic Probability	...	202
6.5	Laws of Probability	...	206
6.6	Conditional Probability	...	213
6.7	Independent and Dependent Events	...	223
	Exercises	...	233
7.	RANDOM VARIABLES		
7.1	Introduction	...	245
7.2	Distribution Function	...	246
7.3	Discrete Random Variable and its Probability Distribution	...	246
7.4	Continuous Random Variable and its Probability Density Function	...	252
7.5	Joint Distributions	...	256
7.5.1	Bivariate Distribution Function	...	256
7.5.2	Bivariate Probability Functions	...	257
7.5.3	Marginal Probability Functions	...	258
7.5.4	Conditional Probability Functions	...	259
7.5.5	Independence	...	259
7.5.6	Continuous Bivariate Distribution	...	263

7.6	Mathematical Expectation of a Random Variable	...	267
7.6.1	Expectation of a Function of a Random Variable	...	270
7.6.2	Properties of Expected Values	...	276
7.6.3	Covariance of Two Random Variables	...	283
7.6.4	Variance of the Sum or Difference of two Random Variables	...	283
7.6.5	Correlation Co-efficient of Random Variables	...	284
7.7	Medians and Modes of Continuous Random Variables	...	286
7.8	Chebyshev's Inequality	...	289
7.9	Moment Generating Function	...	290
7.9.1	Cumulant Generating Function	...	292
7.9.2	Relation between Cumulants and Moments	...	293
7.9.3	Characteristic Function	...	293
	Exercises	...	294

8. DISCRETE PROBABILITY DISTRIBUTIONS

8.1	Introduction	...	303
8.2	<u>Binomial Probability Distribution</u>	...	303
8.2.1	Derivation of Binomial Probability Distribution	...	304
8.2.2	Binomial Frequency Distribution	...	310
8.2.3	Properties of the Binomial Probability Distribution	...	311
8.2.4	The Recurrence Formula for Binomial Distribution	...	315
8.2.5	Fitting a Binomial Distribution to Observed Data	...	317
8.2.6	Moment Generating and Cumulant Generating Functions of the Binomial Distribution	...	318
8.3	<u>Hypergeometric Probability Distribution</u>	...	320
8.3.1	Derivation of Hypergeometric Distribution	...	321
8.3.2	Properties of Hypergeometric Distribution	...	324
8.4	<u>Poisson Distribution</u>	...	327
8.4.1	Derivation of Poisson Approximation to the Binomial	...	328
8.4.2	Poisson Frequency Distribution	...	332
8.4.3	Properties of the Poisson Distribution	...	332
8.4.4	The Recurrence Formula for Poisson Distribution	...	336
8.4.5	Fitting a Poisson Distribution to Observed Data	...	336
8.4.6	Poisson Process	...	338
8.4.7	Moment Generating and Cumulant Generating Functions of the Poisson Distribution	...	339

8.5	Negative Binomial Distribution	...	341
8.5.1	Derivation of the Negative Binomial Distribution	...	342
8.5.2	Properties of the Negative Binomial Distribution	...	344
8.6	Geometric Distribution	...	345
8.6.1	Derivation of the Geometric Distribution	...	345
8.6.2	Properties of the Geometric Distribution	...	347
8.6.3	Moment Generating Function of the Geometric Distribution	...	347
8.7	Multinomial Distribution	...	348
8.7.1	Derivation of the Multinomial Distribution	...	348
	Exercises	...	350
9.	CONTINUOUS PROBABILITY DISTRIBUTIONS		
9.1	Introduction	...	361
9.2	Uniform Distribution	...	361
9.2.1	Properties of the uniform distribution	...	362
9.2.2	Moment-Generating Function	...	362
9.3	Exponential Distribution	...	363
9.3.1	Properties	...	364
9.3.2	Moment-Generating Function of Exponential Distribution	...	364
9.4	Gamma and Beta Distributions	...	365
9.4.1	Gamma Function	...	365
9.4.2	Beta Function	...	367
9.4.3	Gamma Distribution	...	368
9.4.4	Properties of Gamma Distribution	...	368
9.4.5	Moment-Generating Function of Gamma Distribution	...	369
9.4.6	Beta Distribution of the First Kind	...	370
9.4.7	Properties of $\beta_1(m, n)$	...	371
9.4.8	Beta Distribution of Second Kind	...	372
9.4.9	Properties of $\beta_2(m, n)$	...	373
9.5	Normal Distribution	...	373
9.5.1	Standardized Normal Distribution	...	375
9.5.2	Properties of Normal Distribution	...	377
9.5.3	Moment Generating and Cumulant Generating Functions of the Normal Distribution	...	384
9.5.4	Tabulated Areas of Normal Distribution	...	387
9.5.5	Inverse Use of Table of Areas under Normal Curve	...	393
9.5.6	Normal Approximation to the Binomial Distribution	...	398
9.5.7	Normal Approximation to the Poisson Distribution	...	403
9.5.8	Fitting a Normal Distribution	...	405
9.5.9	Ordinates of the Normal Distribution	...	408

10.	Exercises	... 411
	SIMPLE REGRESSION AND CORRELATION	
10.1	Introduction	
10.2	Deterministic and Probabilistic Relations or Models	... 421
10.3	Scatter Diagram	... 422
10.4	Simple Linear Regression Model	... 422
10.4.1	An Aside - The Principle of Least Squares	... 423
10.4.2	Least-Squares Estimates in Simple Linear Regression	... 425
10.4.3	Properties of the Least-Squares Regression Line	... 430
10.4.4	Standard Deviation of Regression or Standard Error of Estimate	... 430
10.4.5	Co-efficient of Determination	... 432
10.5	Correlation	... 434
10.5.1	Pearson Product Moment Correlation Co-efficient	... 434
10.5.2	Correlation and Causation	... 436
10.5.3	Properties of r	... 437
10.5.4	Correlation Co-efficient for Grouped Data	... 439
10.6	Rank Correlation	... 443
10.6.1	Derivation of Rank Correlation	... 443
10.6.2	Rank Correlation for Tied Ranks	... 445
10.6.3	Co-efficient of Concordance	... 447
	Exercises	... 449
11.	MULTIPLE REGRESSION AND CORRELATION	
11.1	Introduction	... 457
11.2	Multiple Linear Regression With two Regressors	... 458
11.2.1	Expression of Multiple Linear Regression in Deviation Form	... 460
11.2.2	Standard Error of Estimate	... 462
11.2.3	Co-efficient of Multiple Determination and Multiple Correlation	... 463
11.2.4	Subscript Notation	... 464
11.2.5	Properties of Residuals	... 465
11.2.6	Multiple Regression in terms of Linear Correlation Coefficients	... 466
11.3	Multiple Correlation Co-efficient	... 468
11.4	Partial Correlation	... 471
11.4.1	Relationship between Multiple and Partial Correlation Co-efficient	... 477
11.5	Curvilinear Regression	... 477
	Exercises	... 478

12.	CURVE FITTING BY LEAST SQUARES	
12.1	Introduction	... 485
12.2	Approximating Curves and the Principle of Least-Squares	... 485
12.2.1	Fitting a Straight Line	... 486
12.2.2	Fitting of a second degree Parabola	... 488
12.2.3	Fitting of Higher Degree Curves	... 491
12.2.4	Change of Origin and Unit	... 492
12.3	Exponential Curves	... 493
12.4	Other Types of Curves	... 497
12.4.1	Modified Exponential Curve	... 497
12.4.2	The Gompertz Curve	... 498
12.4.3	The Logistic Curve	... 499
12.4.4	The Makeham Curve	... 499
12.5	Criteria for a Suitable Curve	... 500
12.6	Finding Plausible Values by LS Method	... 500
	Exercises	... 500
13.	TIME SERIES ANALYSIS	
13.1	Introduction	... 511
13.2	Components of a Time Series	... 512
13.2.1	Secular Trend	... 513
13.2.2	Seasonal Variations	... 513
13.2.3	Cyclical Fluctuations	... 514
13.2.4	Irregular or Random Variations	... 514
13.3	Time Series Decomposition	... 515
13.4	Analysing the Secular Trend	... 515
13.4.1	The Method of Freehand Curve	... 515
13.4.2	The Method of Semi-Averages	... 516
13.4.3	The Method of Moving Averages	... 518
13.4.4	The Method of Least-Squares	... 523
13.5	Detrending	... 528
13.6	Analysing the Seasonal Variations	... 529
13.6.1	The Percentage of Annual Average Method	... 530
13.6.2	The Ratio-to-Moving-Average Method	... 531
13.6.3	The Ratio-to-Trend Method	... 533
13.6.4	The Link-Relative Method	... 536
13.7	Deseasonalization of Data	... 538
13.8	Analysing the Cyclical Variations	... 539
13.9	Analysing the Irregular Variations	... 540
13.10	Forecasting	... 541
13.10.1	Forecasting by Exponential Smoothing	... 541
13.11	Serial Correlation	... 542
	Exercises	... 544
	Answers to Exercises	... 553
	Tables	... 573
	Index	... 579